Hall Ticket Number:

Code No. : 21801

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

M.E. (Mech. Engg.: CBCS) I-Semester Main Examinations, January-2018

(Advanced Design & Manufacturing)

Mathematical Methods for Engineers

Time: **3** hours　　　　Max. Marks: **60**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

1. If $\vec{F}=2x\left(y^2+z^3\right)\vec{i}+2x^2y\vec{j}+3x^2z^2\vec{k}$, then find Div(Curl $\vec{F}$).
2. Define Curl of a vector point function.
3. Define Gradient of a scalar point function in tensor fields.
4. Write in full form of $a_{rs}x^s=b_r(r,s=1,2,3,4----n)$.
5. If $\begin{bmatrix} a \\ 1 \end{bmatrix}$ is an Eigenvector of $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, find a.
6. Write the condition for consistency of system of equations $A_{m\times n}X_{n\times 1}=B_{m\times 1}$
7. Find the Laplace transform of $\frac{1-\cos t}{t}$.
8. Define periodic function of Laplace transform.
9. Classify the Partial differential equations.
10. If f(x) = x, $0 < x < 2\pi$ then find a_2.

Part-B (5 × 8 = 40 Marks)

11\. a) Prove that $\nabla\times[(\bar{C}.\bar{r})r^n\,\bar{r}]=(\bar{C}\times\bar{r})r^n$, where $\bar{C}$ is a constant vector and r is magnitude of $\bar{r}$. [4]

b) If ϕ satisfies Laplacian equation, show that $\nabla\phi$ is both solenoidal and irrotational. [4]

12\. a) What is a mixed tensor of second rank? Prove that δ^p_q is a mixed tensor of second rank. [3]

b) A covariant tensor has components $xy, 2y-z^2, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates. [5]

13\. a) Use the triangular factorization (LU factorization) method to solve [4]

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}.$$

b) Solve by Gauss Seidal method $\begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 2 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}$. [4]

14\. a) Using Laplace transformation solve the following differential equation: $\frac{d^2x}{dt^2}+9x=\cos 2t$, if x(0) = 1, $x\left(\frac{\pi}{2}\right)=-1$. [4]

b) Using Convolution Theorem, prove that $L^{-1}\left[\frac{1}{S^3(S^2+1)}\right]=\frac{t^2}{2}+\cos t-1$. [4]

Contd... 2

15. a) A rectangular plate with insulated surfaces is 10 cms wide and so long compared to its width that it may be considered infinite in length. If the temperature along the short edge is given by $u(x,0)=\begin{cases}2x, 0\le x\le 5\\ 2(10-x), 5\le x\le 10\end{cases}$ while the two long edges x = 0 and x = 10 as well as the other short edge are kept at 0^0c. Find the steady state temperature at any point (x,y) of the plate. [6]

b) $\dfrac{\partial^2 V}{\partial x^2}+2\dfrac{\partial^2 V}{\partial x\partial y}+\dfrac{\partial^2 V}{\partial y^2}=0$. Is this equation elliptic, parabolic or hyperbolic? [2]

16. a) Prove that $\nabla\times(\nabla\times\bar{a})=\nabla(\nabla.\bar{a})-\nabla^2\bar{a}$ [5]

b) Express Grad ∅, Div $\bar{f}$ using indices notation. [3]

17. Answer any ***two*** of the following:

a) Find the Eigen values and Eigenvector for the matrix $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$ [4]

b) Find $L^{-1}\left[\dfrac{s^2+s-2}{s(s-2)}\right]$ [4]

c) A string is stretched between the fixed points (0, 0) and (1, 0) and released from rest from the point u(xm0) = A sinn 2px. Find the displacement $u(x,t)$. [4]

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